ANALYSIS OF AIRSPACE TUBE STRUCTURES

Kapil S. Sheth, NASA Ames Research Center, Moffett Field, CA
Tanim S. Islam, UC Santa Cruz, Moffett Field, CA
Parimal H. Kopardekar, NASA Ames Research Center, Moffett Field, CA

Abstract

This paper presents analysis of five airspace tube structures; three of which were previously defined and two new designs that were generated for this research. Five metrics to characterize the performance of these tubes are described. The metrics address the spatio-temporal utilization, frequency and angles at which aircraft cross tubes, along with separation of aircraft with and without tubes. All of the designs were incorporated in a common simulation platform for evaluation. The results indicate that current designs of tubes have low utilization and improvements are needed for additional benefits. Other structural parameters for consideration in future designs are presented along with visualization of a simple three-dimensional tube network.

Introduction

The current National Airspace System (NAS) structure includes twenty Air Route Traffic Control Centers (ARTCCs) within the continental United States, 40 to 80 sub-regions called sectors within each of those ARTCCs, airways and jet routes, and navigational aids [1]. The shape and size of sectors have historically evolved to accommodate smooth traffic flow and surveillance requirements in the airspace to minimize air traffic controller workload while maintaining ability of aircraft to fly with currently available equipment. Structural design of the NAS could strongly influence throughput and efficiency of future traffic flows when demand is predicted to increase significantly [2]. The objective of airspace reconfiguration is to gain efficiency and capacity. Kopardekar, et al. [3] have noted that a future airspace should be flexible and dynamic to accommodate increased traffic, weather constraints and evolving equipage. One of the concepts to achieve the future airspace design for handling increased traffic is the implementation of tubes or corridors-in-the-sky. These could be developed as a new class of airspace or as a part of the existing airspace structure. It is hypothesized that tubes created at appropriate times and locations could help accommodate higher density of traffic due to well organized flows within tubes providing better service to the users. It would also provide reduced workload for the air traffic managers because of transfer of separation responsibility within tubes to the equipped cockpit of aircraft and result in lower traffic outside the tubes. The feasibility and benefits of establishing such airspace tubes are open research topics.

There are several concepts in the literature on how to reconfigure the different airspace structures to accommodate increased traffic demand. Some of them include the redesign of current sectors within each of the twenty Centers [4, 5]. Others propose newer classes of airspace, possibly embedded within the current structure [6]; and some suggest modification through the use of generic airspace [3]. In one of the concepts proposed by Sridhar, et al. [7], the implementation of tube structures is achieved by connecting 22 clusters comprising of the top 500 airports. Klein, et al. [8] proposed a concept which configures the sectors based on traffic flow patterns during bad weather conditions leading to an Airspace Playbook. The Federal Aviation Administration routinely uses the Severe Weather Avoidance Playbook for rerouting of flows around convective weather. The airspace playbook concept adjusts the sectors (combining or splitting or creating new ones) based on the 'play' in effect for various weather and traffic situations. Despite the establishment of several tube concepts, research describing their usefulness through specific performance metrics is not available.

The feasibility and benefits of airspace reconfiguration in the form of tubes or corridors-in-the-sky is analyzed in this study. Three previously designed tube structures by Sridhar, et al. [7], Xue, et al. [9] and Gupta, et al. [10] were used in this research. Based on current airspace structure and traffic flow patterns, two new designs were created for this study. All of these five tube structures were analyzed. These tubes differ based on the objectives of employing the current airspace structure, maximizing the occupancy or optimizing the cost of establishing a tube structure. The utilization of the tube alternative is evaluated by determining the instantaneous and volumetric count of aircraft flying in them for assessing the usefulness of the design. Three other metrics are examined to understand the benefits and feasibility of the tube concept. These metrics are the number of traffic conflicts with and without the tube structure; and the frequency and angle at which aircraft cross the tube. Based on results from a parallel study [9], the maximum excess distance associated with entering and leaving the tube compared to the nominal flight path was kept constant in this research. The process and metrics derived in this study can be used to guide the assessment of other tube structure designs in the common simulation platform used for this research. The purpose of studying this variety of structures was to elucidate structural parameters leading to better tube designs.

The organization of the remaining paper is as follows. Five designs of tubes analyzed in this study are presented followed by the method used for comparing the designs. Five performance metrics are described and the results for each of the five designs are presented. Preliminary conceptual three-dimensional tube structures are discussed and the findings of this research are presented in the last section.

Design of Tube Structures

This section describes various tube structures considered for this analysis. Each of these tube designs was implemented in the Future ATM Concepts Evaluation Tool (FACET), a flexible NAS-wide airspace and air traffic simulation developed at NASA Ames Research Center [11]. The following five tube structures were considered for analysis.

Jet Routes based Tubes (T1)

For the first tube design, the Enhanced Traffic Management System data were used to obtain flight plans for all the flights for Aug. 24, 2005. These filed flight plan strings were parsed to obtain the top 50 jet routes used in the United States, which were incorporated as the first tube structure design. The idea behind the first tube design is that most aircraft fly (especially at higher altitudes) on the current day high altitude jet routes already. This naturally makes it a reasonably good tube design. This jet route based tube design was created for this research and is based on the current jet route structure in the United States. Fig. 1 shows this T1 tube structure in a FACET display in green.

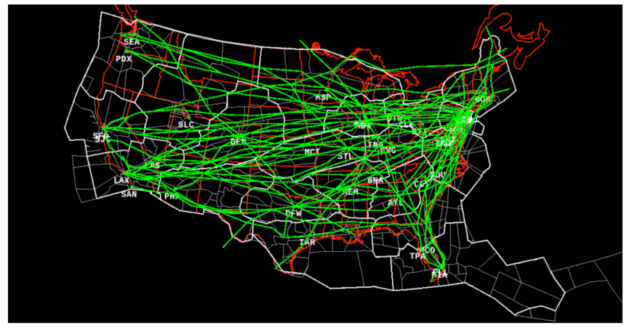

Fig. 1: Top-50 jet routes tube design (T1) in FACET.

Delaunay Triangulated Cluster based Tubes (T2)

In the second tube design, the top 500 airports based on daily operations were clustered using a weighted-proximity technique [7]. The top 22 airports from the list were chosen as seeds around which all other airports were added to form clusters, based on their proximity to the seed airports. The centroids of each of these clusters were chosen as nodes of the traffic network. The resulting network, created using Delaunay triangulation [12] consisting of 55 links and 22 initial airport seeds, was used as the tube design. The idea behind this structure is to capture the airports with most operations, to account for maximum traffic and therefore, high occupancy in the resulting tube network. Fig. 2 shows this T2 tube structure.

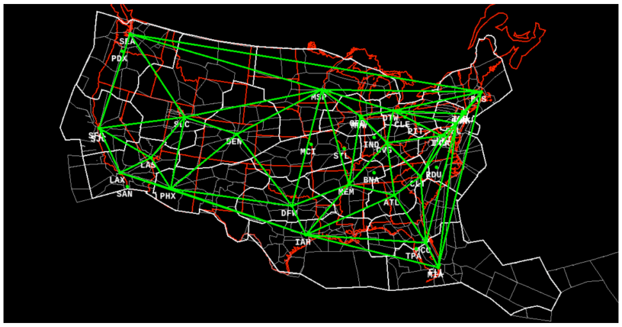

Fig. 2: Delaunay triangulated tube design (T2).

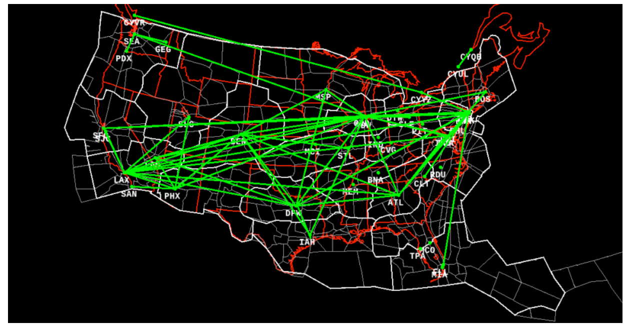

Fig. 4: Traffic density profile based tube design (T3).

Traffic Density based Tubes (T3)

Another new tube design created for this research is a traffic density based tube design, T3. The air traffic for Aug. 24, 2005 was played back through FACET, recording the count of all aircraft on a 10 nmi by 10 nmi grid draped over the continental United States. At the end of the twenty-four hour period, each of the grid cells were color-coded based on the cumulative count of aircraft in each cell. This color-coded grid captured in FACET for a six-hour period from 18-24 UTC (2-8 pm EDT) is shown in Fig. 3. The red cells had high numbers (>90) of aircraft; the blue cells had very few aircraft (<30), while the empty (gray) cells had zero aircraft count. Once this grid was available, the origin-destination information for all the aircraft flying through the highest count red grid cells was obtained. These origin-destination pairs were sorted and the top-50 pairs were chosen as the start and end points of this traffic density based tube structure, T3. Fig. 4 shows this T3 tube design.

Hough Transform based Tubes (T4)

A tube design using great circles developed by Xue et al. [9] was used as the fourth structure for this analysis. All aircraft for Aug. 24, 2005 were flown in a great circle simulation in FACET. For each of the flights, their routes were translated from rectangular to polar coordinate frame of reference defined by a Hough transform. The top 50 maximum density grid cells in the polar reference were selected and improved based on traffic-weighted centroids of the grid cells. These centroids were further optimized based on a genetic algorithm to add other aircraft that could fly on the tubes with no more than 5% relative excess distance of their initial origin-destination great circle distance. Using the radial distance and angle of the refined centroids, 50 great circle trajectories were selected for this tube structure. This T4 great circle tube network (with the circles limited to arcs within the continental US) is represented in Fig. 5.

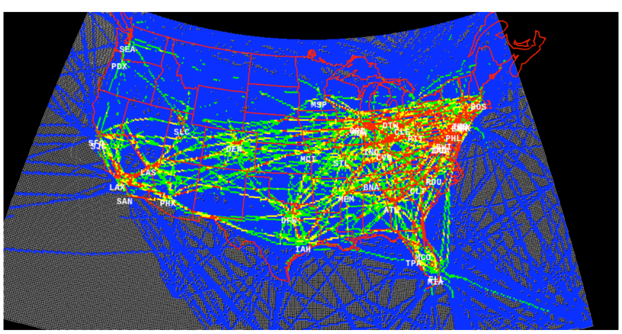

Fig. 3: Total number of aircraft color-coded cells in a traffic density grid.

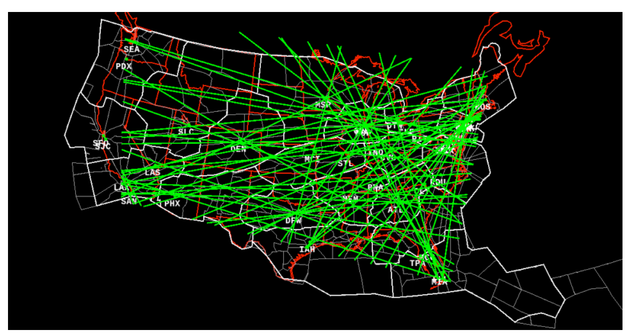

Fig. 5: Great circle tube design (T4).

Network-Flow Cost Optimized Tubes (T5)

The last tube structure considered for this research is presented in Gupta, et al. [10]. Starting from the initial concept study and tube network proposed in Sridhar, et al. [7], a multi-commodity network flow problem was set up. The objective function was the total cost of establishing a network, which included cost of each link and cost of the entire network, along with cost of travel time for each flight. The optimization problem was solved in a mixed integer programming framework to obtain the network of 131 links between 25 airports shown in Fig. 6 in a FACET display.

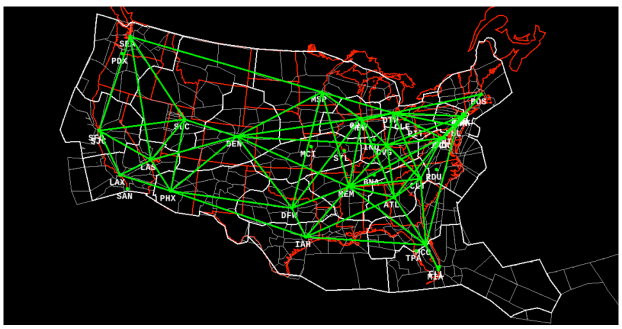

Fig. 6: Network-Flow Cost Optimized tube design (T5).

Comparison Method

The five designs (T1 through T5) described above were implemented in FACET software. The performance of these designs was compared using the same traffic data and five metrics defined in the next section. Even though the analysis was performed for 24-hours of Aug. 24, 2005 in four six-hour blocks, only results for the 18-24 UTC (2-8 pm EDT) corresponding to the highest traffic counts in the National Airspace System are presented here. The total number of aircraft during that time period considered in this analysis was 22,288.

The aircraft traffic was considered from all altitude levels (above 12,000 ft only) and included in the tube. Hence, the tubes encompassed all flight altitudes above 12,000 ft but more importantly, were considered to be two-dimensional in nature. For this analysis, precise temporal and spatial locations where aircraft could join and leave (entry and exit points) tubes were not specified. At times when an aircraft could travel in a tube, it would enter and exit the tube orthogonally. Additionally, only domestic flights within the United States were considered and international flights were removed from the data.

In Ref. 9, an excess distance parameter was considered. This parameter provides the overhead associated for operators of flights and is helpful in deciding the formation and location of tubes. It was found that when only the aircraft that fly on the links of the tubes are considered, the aircraft count (as defined in the utilization parameter described below) is quite low. However, the utilization increased significantly when all aircraft in the vicinity of the tubes were considered to be using the tube when their excess distance to fly on the tube was not larger than 5% of their nominal origin-destination great circle distance. As this excess distance parameter was increased up to 10%, the number of aircraft using the tubes increased as well, but the aircraft flew farther to get to their destination. In order to contrast the performance of each of the designs with the common assessment platform in FACET, this excess parameter was set at 5% for simulating each of the tube structure traffic.

Performance Metrics and Results

Five performance metrics for comparison of tube structures are defined in this section: the instantaneous occupancy and volume occupancy factors of tube utilization, separation parameters, tube crossing angles and frequency. The results for each tube alternative are simultaneously presented below corresponding to each of the metrics.

Utilization of Tubes

Instantaneous Occupancy

The instantaneous occupancy parameter, U_1, presents how many aircraft are flying in the tube compared to the total number of aircraft at the same instant in time. This metric represents the temporal utilization of tubes. For each of the tube structures, this parameter was calculated as follows:

$$U_1(t) = \frac{n_s(t)}{n_a(t)} \qquad (1)$$

where n_s is the number of aircraft on the tube structure and n_a is the total number of aircraft in flight at the same instant in time, t. Plots for utilization for all five tubes are presented in Fig. 7, with blue, green, red, black and purple lines corresponding to each of the T1 through T5 tube designs, respectively. From the figure, the network designs T5 and T4 appear to have highest instantaneous occupancy. An interesting feature observed from the time varying utilization plot is that based on demand, one can determine the trigger times of when to invoke the tube and when to turn it off.

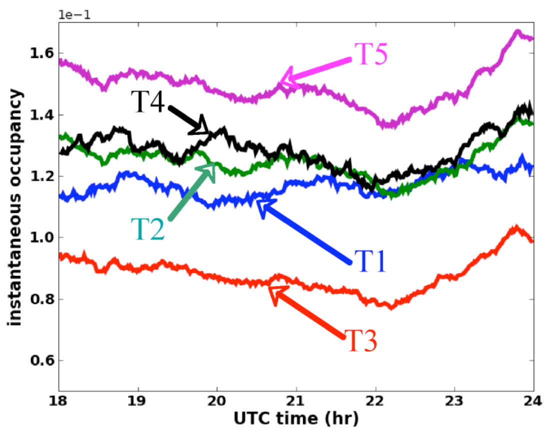

Fig. 7: Instantaneous occupancy parameter.

In order to see the nominal behavior of the tubes, the average instantaneous occupancy parameter, $\overline{U_1}$, was computed for each of the designs as defined in Eq. 2, where Δt is the time step at the evaluation instance of time, t. The step Δt is retained only to maintain functional form of the discretized integral. Again, T5 and T4 performed best with that metric, as seen from the results presented in Table 1.

$$\overline{U_1} = \frac{\sum_t \left(n_s(t) * \Delta t\right)}{\sum_t \left(n_a(t) * \Delta t\right)} \tag{2}$$

Volume Occupancy

The utilization can also be defined alternately in terms of the total number of aircraft that can possibly occupy each tube. This metric represents the spatial or volumetric utilization of tubes. This volume occupancy parameter, U_2, is defined as follows:

$$U_2(t) = \frac{n_s(t)}{N} \tag{3}$$

where n_s, as defined before, is the number of aircraft in the tube structure at an instant in time, and N is the maximum number of aircraft that can simultaneously occupy the tube. Thus, U_2 defines how full the tubes are compared to the total space available in them. N for each tube is computed based on a 5 nmi separation between each aircraft, and the length of that tube for 31 altitude values starting from 12,000 ft to 42,000 ft in 1,000 ft increments. These numbers for each tube design are presented in Table 1. Fig. 8 shows the volume occupancy curves for each of the tubes with the same colors as in Fig. 7. It is quite clear that T5 performs extremely well having large volume occupancy numbers, with T2 following closely. The rest of the designs appear to have very long tube

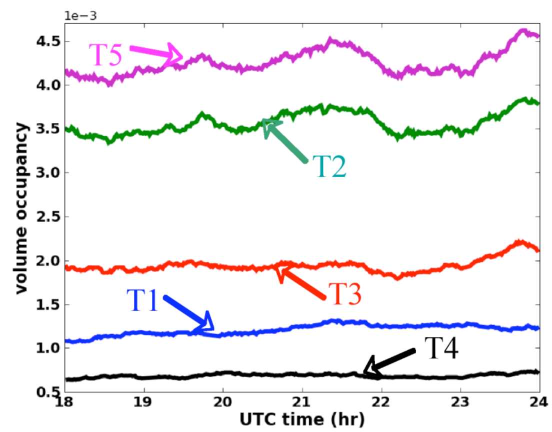

Fig. 8: Volume occupancy parameter.

structures and even though more aircraft may fly on them (e.g. T4), the effective volume occupancy is relatively low. This metric addresses the issue of how long a tube should be for the cost associated with not just establishing but maintaining the entire tube structure, since many aircraft may fly on a tube but most may occupy small segments of it.

The average volume occupancy, $\overline{U_2}$, was computed as shown (Eq. 4):

$$\overline{U_2} = \frac{\sum_t \left(n_s(t) * \Delta t\right)}{N * T} \quad (4)$$

where Δt is the time step at the evaluation instance of time, t. The total evaluation period, T, is six hours for the results reported here.

From the values in Table 1 it is seen that, for the instantaneous occupancy metric ($\overline{U_1}$), T5 and T4 performed well while for the volume occupancy ($\overline{U_2}$) of the entire tube structure (total length and all altitudes), T5 and T2 seem to perform the best. The $\overline{U_2}$ metric implies that these two tube structures are most full on average with respect to the available space within them.

Table 1: The values of average instantaneous occupancy ($\overline{U_1}$), maximum number of aircraft possible on each tube (N) and average volume occupancy ($\overline{U_2}$) for each tube design.

Metric	$\overline{U_1}$	N (*0.001)	$\overline{U_2}$ (*100)
T1	0.12	492.8	0.12
T2	0.12	177.7	0.36
T3	0.09	228.9	0.19
T4	0.13	955.4	0.07
T5	0.15	177.0	0.42

Separation Parameter

Another parameter studied is the number of conflicts with and without the presence of the tubes. It is presumed that the number of aircraft traveling within the tubes will have additional equipment for guidance within tube structure and/or autonomous flight rules for self-separation. Thus, it is desirable if the number of conflicts significantly reduces in ambient (non-tube) traffic due to the existence of tube structures. The reduced conflicts contribute towards a lower controller workload, not accounting for the incremental complexity of managing traffic in the tubes, if any.

In order to address this parameter for each of the designs, the Chicago, Cleveland, New York and Washington, DC centers were selected due to the high volume of traffic and observed number of conflicts in simulated traffic. There was no controller input in these simulations. Conflict between aircraft is defined as a violation of 5 nmi separation in the horizontal direction and 1000 ft separation in the vertical dimension. Fig. 9 presents the results of the calculation. All the colors representing the tubes remain the same as before but the gray line (labeled Ambient) represents number of conflicts when no tube structure is present. From the plot, it is clear that T4 fares the best with overall lower conflicts, however, T5 and T3 tubes are close behind.

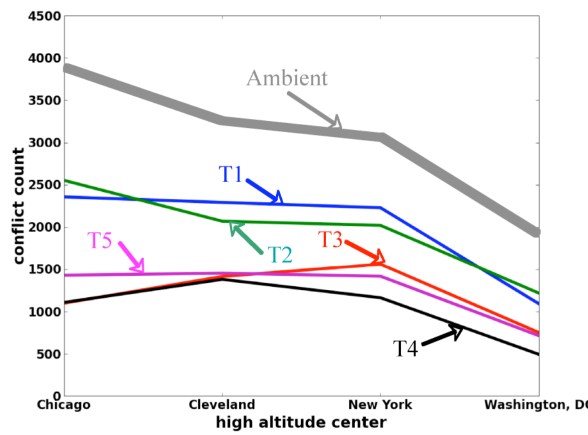

Fig. 9: Conflicts with and without the tube structures.

Tube Crossings and Frequency

While the tube structure may help alleviate the air traffic controller's workload by reducing conflicts, the encounters between the tubes and the remaining (non-tube) traffic must also be considered. Fig. 10 shows the convention used for determining the crossing angle θ. The two-dimensional tubes were always considered oriented from west to east and each tube segment is considered horizontal at the time of aircraft crossing. Each aircraft path is measured with the angle θ considered positive counterclockwise, with respect to the tube plane. With this in mind, the frequency of crossings and crossing angle (θ)

parameters are calculated. The frequency parameter describes how many aircraft intersect this tube structure at any altitude at or above 12,000 ft as a function of time and is represented in Fig. 11 with the same colors for tube designs as before.

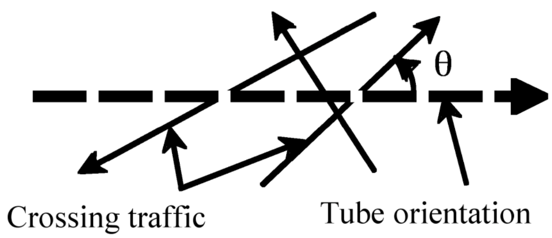

Fig. 10: Convention to define traffic crossing tubes.

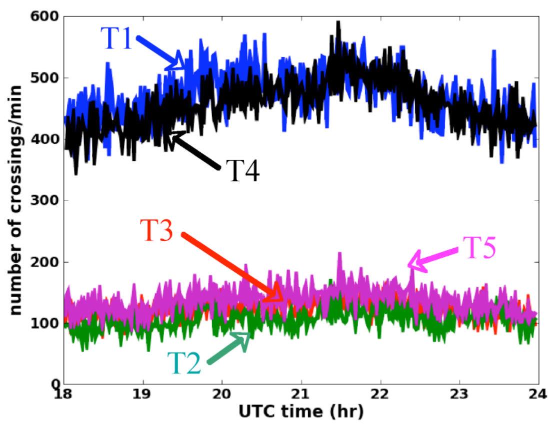

Fig. 11: Number of aircraft crossing as a function of time.

The encounter angles for crossing traffic are calculated to understand the complexity of flow patterns. The density, $\rho(\theta_5)$, with θ_5 representing a five degree bin, is calculated as follows (Eq. 5):

$$\rho(\theta_5) = \frac{\sum_t \left(C(t,\theta_5) * \Delta t\right)}{5 * \sum_{\theta_5} \sum_t \left(C(t,\theta_5) * \Delta t\right)} \tag{5}$$

where $C(t, \theta_5)$ is the count of aircraft at time t within the 5-degree bin, θ_5. The tube encounter angles are shown as a histogram with 5-degree bins in Fig. 12. It is seen from Fig. 11 that T1 and T4 have high volume of crossing traffic, about four times, compared to T5, T2 and T3. In the absence of automation, a high volume of crossing traffic would imply higher controller workload, since the controller would have to separate this crossing traffic from the tube structure. From Fig. 12 it is observed that T1 and T3 have higher instances of head on (180 deg) or in-trail (0 or 360 deg) encounter angles compared to the other tubes. In general controllers prefer the zero degree and 180 degree encounter angles since it is easier to visualize and manage them compared to aircraft crossing at other angles.

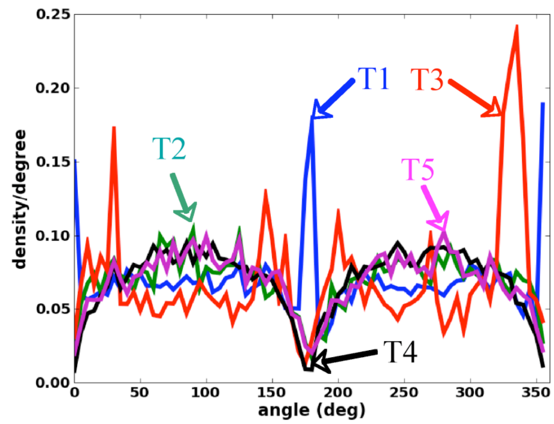

Fig. 12: Histogram of density of tube crossing angles.

Discussion

In Sridhar et al. [7], seven research topics were presented for feasibility and benefits of establishing tube structures. The selection of high-density regions for tube design is the first of those seven and was a topic of that study. This research has attempted to address items corresponding to connectivity, complexity and benefits of tube networks, respectively. The connectivity aspects are addressed by considering five tube designs created by using different objective functions. The complexity and benefits are analyzed through five quantitative performance metrics. However, additional work is needed to address these issues satisfactorily.

Results for utilization, number of conflicts and tube crossings, have been obtained for all five tube structures using current day traffic flow patterns. The results show that the current designs represent mixed performance with respect to the metrics evaluated. The network-flow cost optimized tube design (T5) and Hough transform tube design (T4) have better instantaneous utilization, while T5 and Delaunay triangulated cluster tube design (T2) have better volume occupancies. All three designs T4, T5 and traffic density tube design (T3) have better separation characteristics compared to jet route tube design (T1) and T2. While T1 and T4 have a relatively large amount of crossing traffic, T1 and T3 have good properties of crossing angles from a controller's perspective. The tube design T4, however, is difficult to implement from a pilot's perspective since a continuous path along a great circle is not flyable with current equipage on aircraft, however, in the future it may be possible. The search of a design that performs well with all the metrics is ongoing but will require more research to discover.

As mentioned earlier, the analysis performed in this research did not isolate the traffic based on different altitude levels but considered it in an aggregate manner. In order to conceptualize the three-dimensionality of the tube structures, a smaller tube network for six airports (from tube design T5) in plan-view display is shown in Fig. 12 and in three-dimensional perspective view in Fig. 13. This shows a preliminary representation of tube structures using FACET. In Figs. 12 and 13, only single flight level tubes are shown, but in Fig. 14, the side-view of five-altitude level (24,000 ft in cyan through 40,000 ft in green with 4,000 ft increment) tubes for the same set of six airports are shown. The corresponding perspective view of the multi-layered tube is shown in Fig. 15.

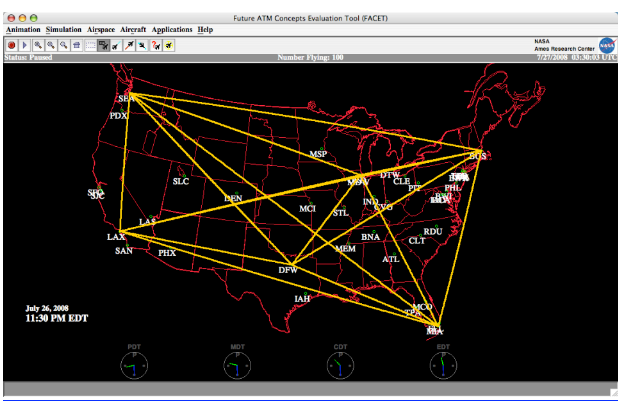

Fig. 12: Plan view of a six airport tube design in FACET.

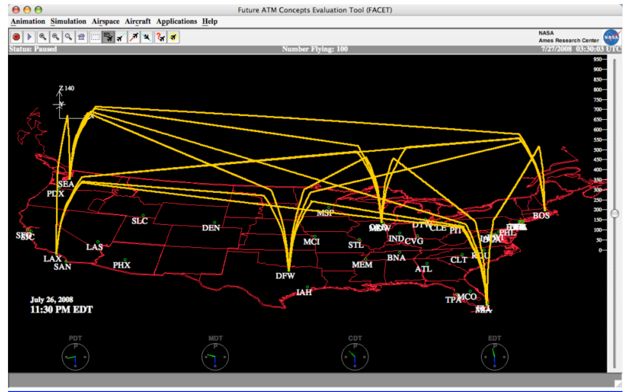

Fig. 13: Perspective view of a six airport tube design.

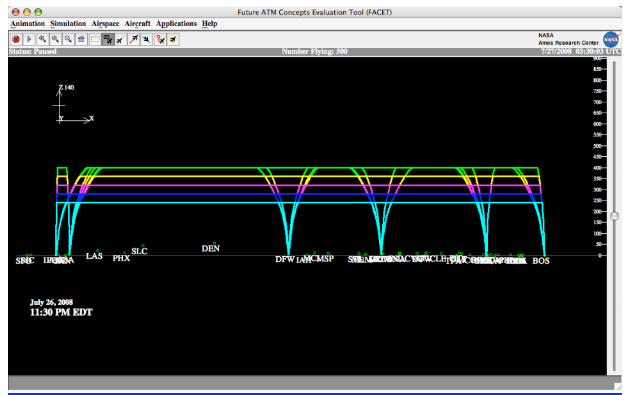

Fig. 14: Altitude view of a six airport tube design with five altitude levels.

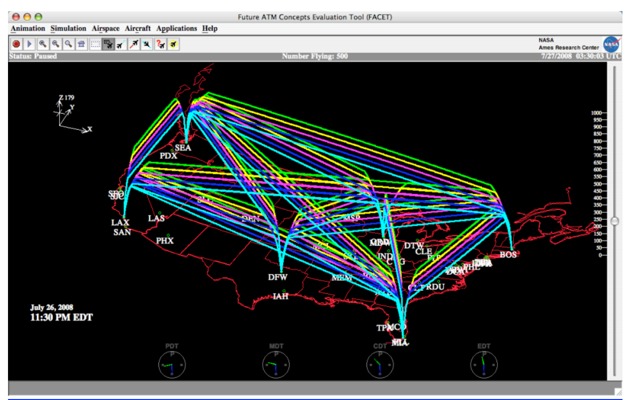

Fig. 15: Perspective view of a six airport tube design with five altitude levels.

The altitude structure described in Figs. 14 and 15 are not used for the design or analysis in this research but is shown here for representation and further analysis. These figures explain the need to consider altitude as an important factor in the design of tubes of the future. It becomes clear that not only aircraft crossing the tubes are an issue but also if many tubes are present, crossing of tubes (by aircraft and other tubes) at various altitudes and the intersection of these tubes with severe weather contours, becomes an implementation problem as well. The volume occupancy for spatial utilization presented earlier was a starting point to describe the cross-sectional structure within each of these tube segments. There could be five lanes of traffic at three different levels within each segment of the tube, but more details on the structural parameters need to be researched.

Two additional issues that need to be accounted for in the creation of new tube networks of the future, are increased traffic estimates and flight routing. This research was conducted with current traffic demand but performance metrics for a future traffic demand of 1.5 to 3 times current demand need to be evaluated as well. Also, in the future, it would be reasonable to assume that users of the airspace will be able to file and fly wind optimal routes for each flight. At the current time, the equivalent of T3 for a wind optimal route structure has been developed using the traffic density grid and will be presented in a future report. A traffic scenario of the future with approximately three times the current traffic levels has been developed and will be reported in that study as well.

The process of traffic joining and leaving tubes is another important aspect of the tube structure definition outlined in Ref. [7] and can be considered in conjunction with the variation of the relative excess distance parameter. The times when entry into or exit from tubes is allowed, or the physical locations where such actions are permitted, were not considered in this study and are still open research topics. The current tube structures analyzed in this paper also did not account for the start and end points of tubes and assumed they are available for occupation throughout its defined length.

Conclusion

The airspace structure of the future will need to be flexible and dynamic to accommodate forecasted increase in traffic demand. Several issues in the design of tubes have been outlined here as well as in previous studies. A Delaunay triangulation method, a Hough transform technique and a network-flow cost optimization framework were previously used to design tubes. Two newly created designs using jet routes and a traffic density profiler, along with the three earlier designs, were analyzed in this study. A common platform for analyzing tube structures was developed, which can be used for qualifying future designs. Five performance metrics of instantaneous and volume occupancy, separation parameters and tube crossing angles and frequency were used to assess the feasibility and benefits of tube designs. Although the five structures considered here did not present a single tube structure that was outstanding, several important aspects of future tube designs were highlighted as a result. Based on the analysis, it was learned that a good tube structure design should have high utilization (instantaneous and volume) and lower along-track encounters with other traffic while reducing conflicts in non-tube traffic. It appears that the network-flow cost optimized tube design (T5) performed better overall.

It was determined that incorporating altitude for tube designs within future traffic scenarios and future flight routing patterns is important and should be accounted for in newer tube network design methods. In the future, it would be good to obtain a weighted metric which combines each of the five metrics considered here (and any others that are considered important subsequently) for a common assessment of various tube structures.

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Email Addresses

Kapil.Sheth@nasa.gov

Tanim.S.Islam@nasa.gov

Parimal.H.Kopardekar@nasa.gov